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Kriging, cokriging or stochastical simulations, and the choice between deterministic or sequential approaches

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ABSTRACT

The data presented here for the first time in the Croatian geomathematical community, aim to establish several criteria which will aid in the selection of deterministic or stochastic geostatistical methods of estimation.
This review associates the theoretical background of kriging, cokriging or stochastic simulations with some results obtained by mapping Badenian clastic reservoirs in the Drava Depression. The selected reservoirs are located in the Stari Gradac-Barcs Nyugat field, in the Western part of the Drava Depression, and at the Beničanci field in the Eastern part of this depression.

Both datasets (each with 14 points) include mean porosity values taken from well log analysis in the reservoir interval at the well sites. This resulted in significant uncertainties in the variogram models, especially with the determination of range (at the secondary variogram axis).

The critical advantage was the availability of a seismic attribute that could be used as a secondary and co-regionalized variable (porosity is the primary variable). In the first example (the Beničanci field) the seismic attribute was correlated with the average logged porosities. This made it possible to apply the cokriging method as the best interpolation option. The cross-validation result was 2.19 for 14 wells.

In contrast, the existence of only a primary variable at the Stari Gradac – Barcs Nyugat field, forced the application of stochastic simulations as the better estimation tool, which can better describe the porosity changes in inter-well areas.

Keywords: kriging, cokriging, Gaussian simulations, porosity, Beničanci field, Stari Gradac – Barcs Nyugat field, Badenian, Drava Depression, Croatia

1. INTRODUCTION

Geostatistics is a well-established tool in different geosciences, primarily defined as advanced interpolation algorithms. In most cases, geostatistical methods offer several advantages compared to other mathematically simpler interpolation techniques. The most important advantage of geostatistics is the variogram function itself. There are several spatial interpolation methods that have the ability of giving weights to data points according to their relative position to the location to be estimated. However kriging is the only method that can take into consideration the spatial anisotropy of the parameters being estimated through the anisotropy of direction variogram, and, what is more, without the variogram it would be nearly impossible to estimate a geological attribute which does not have finite variance like fracture porosity (János Geiger, personal communication, 2007).

Moreover, the variogram as a basic geostatistical term forces accurate determination of spatial relationships among hard-data. Of course, it is not a magic tool, but an experienced geologist can very precisely define the weighting coefficients between hard-data and their influence at the new estimation point.

The geostatistical idea was mostly developed independently of major achievements in the field of spatial statistics. The world-renowned pioneer work was done by KRIGE (1951). The first widely known results are those in the works of Mathéron (e.g. MATHÉRON 1962, 1963, 1965). These included detailed explanations of the kriging, as a basic geostatistical technique equivalent to the method of mean squares applied in a linear Gaussian model. These papers were followed by many others from geostatistical experts that improved theory and practice of different geostatistical methods. A very famous book was written by JOURNEL & HUJIBREGTS (1978). Andre Journel was Matheron's student and he established geostatistical courses at Stanford University. Moreover, RIPLEY (1981) showed the link between the mean squares method and the linear Gaussian model. Ten years later CRESSIE (1991) described geostatistics as one of the three main fields in spatial statistics. ISAAKS & SRIVASTAVA (1989) published their well accepted easy-learning geostatistics book for reservoir characterization. DEUTSCH & JOURNEL (1997) produced their geostatistical library for the most famous geostatistical program GSLIB. There are a lot of other excellent books and papers.

Geostatistics received very strong input from modern reservoir characterization in the nineteen-eighties. These results improved our knowledge and visualization about the subsurface, especially in the inter-well area.

Well and seismic data can be applied in geostatistical models of any kind (mapping, stochastic or indicator simulations). In general, geostatistical results can be divided into two groups – deterministic and stochastic. The geological model can be defined as deterministic if the same inputs always give the same output, e.g. for the same dataset, using identical interpolation methods, the inter-well area will be mapped consistently each time. The model is stochastic if the inter-well area, especially far away from hard-data, is described by similar but different solutions each time. Hypothetically, the random models in each case will yield different results, but this is totally useless and can be considered only as a possibility in unconditional simulations. According to the classification of JENSEN et al. (2000) reservoirs can be considered as:

1. Completely *deterministic reservoirs* where the inter-well area is well known, correlated and described by palaeoenvironments. However, such localities are rare, and mostly include mature fields, with a lot of wells. These are of interest only in the development stage or when searching for »by-passed« oil;
2. *Stochastic reservoirs* are those that include some uncertainties, but where the architecture is mostly known at the macro-scale. However, the inter-well area cannot be described for certain, and improved interpolation methods, like geostatistics, are very desirable.
3. *Random reservoirs* are very rare and are mostly not a goal of geological modelling. There are only several wells and/or gravimetric and magnetometric data that could be described by Monte Carlo sampling or by analogy with similar prospects.

The geostatistical deterministic approach includes different kriging and cokriging methods. A stochastic approach also encompasses different methods that can represent the reservoir through a set of realizations. For the most part, the stochastic approach can be more descriptive than deterministic methods. However, there is also a second additional source of information, which can improve any reservoir picture whether the deterministic or stochastical approach has been used. These are seismic attributes. Cokriging is a deterministic approach that uses seismics as a secondary source of information, describing the inter-well area in more detail than has been possible using kriging. Stochastic simulations based on cokriging as zero-realization, also use seismic attribute(s) as secondary sources of information. The constant »problem« is the reliability of seismic inputs, i.e. whether they correlate with well data or just show a trend.

Several papers recently published in Croatian journals, introduced geostatistical theory to the Croatian geological community, and show the different methods applied at several hydrocarbon fields. In his dissertation, MALVIĆ (2003), used data from the Bjelovar subdepression and analyzed it by experimental variograms. MALVIĆ & ĐUREKOVIĆ (2003) compared the results of porosity mapping in limited input datasets obtained by three different methods – Inverse Distance Weighting, Ordinary Kriging and Collocated Cokriging. The summary results of porosity mapping of the entire western part of the Drava depression (Molve, Kalinovac and Stari Gradac) were published by MALVIĆ (2005). Finally, the geostatistical approach had been connected to depositional models and porosity distribution, (MALVIĆ, 2006). The first stochastic approaches were applied by SMOLJANOVIĆ & MALVIĆ (2005).

The ever present and legitimate engineering and scientific question is, which approach is more appropriate – the deterministic like kriging or cokriging or the stochastic that varies hard-data in statistically allowed limits. The aim here is to either answer this question, or provide a means of assessing which would be the best choice.

2. KRIGING AND COKRIGING THEORY

The theory of variogram analysis has been explained in many papers and books, and represents the necessary input for kriging and cokriging interpolation methods. Determination of spatial dependence is mostly done by a variogram function calculation (**Equation 1.0**):

$$2\gamma\left(\vec{h}\right) = \frac{1}{n} \cdot \sum_{i=1}^{n}\left[z\left(\vec{x}_i\right) - z\left(\vec{x}_i + \vec{h}\right)\right] \quad (1.0)$$

Where are:

$2\gamma\left(\vec{h}\right)$ – *variogram*

n – number of data pairs compared on h distance

$z\left(\vec{x}_i\right)$– *variable value on chosen location (x_i)*

$z\left(\vec{x}_i + \vec{h}\right)$ – *variable value on the location with »h« distance from the initial location (x_i+h)*

Variogram value depends only on the spatial distribution of locations, e.g. on the number of known values over a chosen distance. The results are experimental variogram curves, described with theoretic models, which are the input for geostatistical estimation methods. The basic geostatistical estimation theory is called **kriging**, which can be extended to cokriging and stochastic methods. In general, kriging techniques are called the best linear unbiased estimators (abbr. BLUE), except for Simple Kriging which is only the best unbiased estimator. The *best* shows that weighting coefficients are calculated from minimum kriging variance, *linear* means that estimation is done by linear combination of hard-data, *unbiased* estimation assumes that expectation value is true for the entire imaginary population and *estimation* is methodology. The general kriging formula is shown in **Equation 1.1**:

$$z_k = \sum_{i=1}^{n} \lambda_i \times z\left(\vec{x}_i\right) \qquad (1.1)$$

The regionalized variable is estimated at location *(Z_k)* from existing hard-data $Z\left(\vec{x}_i\right)$ at location $\vec{x}_i$ accompanied by a true weighting coefficient *(λ_i)*. It also means that input data are characterised by a Gaussian distribution (which is often not the case, and data are transformed in a Gaussian-like or standard normal probability distribution). Final kriging values are also shown by a normal distribution. This simple formula given in Eq. 1.1 can be extended in kriging matrix equations that can be symbolically shown as **Equation 1.2**:

$$[W] \times [\lambda] = [B] \qquad (1.2)$$

This is simple matrix formula that describes all kriging techniques like *Simple Kriging, Ordinary Kriging, Indicator Kriging, Universal Kriging, Disjunctive Kriging* and others. The calculation of the weighting coefficient is the main task of the kriging equations. The higher values will be calculated for the hard-data located at the principal axis of the anisotropy. These coefficients also can be negative, but the condition $\sum \lambda = 1$ is satisfied for Ordinary Kriging that is used in this analysis.

Moreover, variogram analysis can pointed at geological elements like structural axes, depositional channels etc. The weighting coefficient values finally depend only on distances among hard-data included in an ellipsoid of spatial dependence. However, advantages of geostatistics cannot be confirmed in cases of very limited input datasets. Such a limit can be set at 10 or 15 hard-data points, because smaller datasets cannot be described by a variogram model.

Cokriging, like kriging, also includes several estimation techniques, mostly identical kriging techniques (Simple Cokriging, Ordinary Cokriging etc.). The only condition is the introduction of a secondary variable (or co-variable) that is: (a) in meaningful connection with the primary variable, (b) sampled at many locations and (c) significantly correlated with the primary variable. The mathematical extension is very simple, derived from Eq. 1.1 and is given in **Equation 1.3**:

$$Z = \sum_{i=1}^{n_z} \lambda_i \cdot z\left(\vec{x}_i\right) + \sum_{j=1}^{n_w} \chi_j \cdot w\left(\vec{y}_j\right) \qquad (1.3)$$

Where:

$Z = \sum_{i=1}^{n_z} \lambda_i \cdot z\left(\vec{x}_i\right)$ = *the part applied to the primary variable,*

$\sum_{j=1}^{n_w} \chi_j \cdot w\left(\vec{y}_j\right)$ = *the part applied to the secondary variable.*

Cokriging methods and co-regionalized variables are related in the same way as the theory of regionalized variables and kriging. Coregionalization describes the spatial variation of several variables. This is extended from the theory of kriging and a regionalised variable to several cokriging variables. These variables have a multivariate spatial cross-correlation as well as the univariate spatial auto-correlation. They are then called co-regionalized variables and can be used to understand spatial processes that manifest themselves as several measured variables (ROSSITER, 2005). Two or more dependant variables can also be interpolated from collocated observations using Regression Kriging or Kriging with External Drift, but this is not discussed here. More often there are more observations of the co-variable, i.e. at some points where the target variable was not measured and requires coregionalization. A typical example is seismic signals received from lithological boundaries and sections of a hydrocarbon reservoir.

Coregionalization is a general term for the theoretical model of how several variables spatially co-vary. The idea is that the process that gives rise to one variable is the same, or at least related, to the process that is manifest in the other variables. All the variables involved are regionalized variables and in addition they are related both in features and geographic space, hence they are co-regionalized. The procedure of coregionalization can be reflected in the variogram calculation, which means that there could be two types of variograms modelled: (a) directional or omnidirectional variograms calculated for a single regionalized variable, i.e. one variogram per variable (see Eq. 1.1) or (b) cross variograms calculated for the pair of regionalized variables e.g. as described in ROSSITER, (2005).

From **Equation 1.3**, determination of weightings for λ (the target variable) and χ (for the co-variable), minimizes the prediction error. It is why cokriging can be described, like kriging, by the acronym BLUE (*best linear unbiased estimator)*.

This short review of deterministic geostatistical tools is only a minimal summary necessary for understanding the research described in this paper. More details can be found in many publications including GOOVAERTS (1997), HOHN (1988), ISAAKS & SRIVASTAVA (1989), JOURNEL & HUIJBREGTS (1978), LIEBHOLD et al. (1993), XU et al. (1992) and others.

3. THE CONCEPT OF STOHASTIC SIMULATIONS

Simulations, writing literally, preserve the flavour of real subsurface variability, mapping all possibilities between the extreme values of the regionalized variable. If simulation is »conditional«, it honours the observed data exactly, without the attendant smoothing of the interpolated estimates, as in kriging. Simulation provides visual and quantitative measures of the uncertainty of the estimated variable (»cosimulation«, analogous to cokriging). Data are honoured where the underlying trend or pattern is reproduced (e.g. as in kriging), whereas data are reproduced where predicted values are forced to be equal to observed values.

There is, of course, a difference between simulations and kriging. Kriging produces just one map of estimates which is

»best« (and hence unique) in some statistical sense, but which does not reproduce global statistics (histogram, variance, covariance). Kriging is a global estimator, and its estimate represents all the data within a defined variogram ellipsoid.

The kriged grid honours data point-values only if the particular point coincides with a grid node. The kriging map is useful to show smooth variations and underlying trends, but not the inherent local variability of the data. Kriging provides an incomplete measure of local accuracy (i.e. kriging »variance« may not represent a Gaussian distribution of errors), and no measure of joint accuracy when several locations are considered together. All these facts are also valid for cokriging techniques.

The simulation is also a local estimator. The goal is the reproduction of global patterns of spatial continuity and global statistics (histogram, covariance), rather than local accuracy. Simulation is used to demonstrate the nature of local variability and global patterns of local variability, such as the connections and trends among differentially valued regions. The simulation produces any number of statistically equivalent maps which, when taken together, define the local estimation uncertainty as well as the global pattern of uncertainty. Undertaking simulation, several approaches can be selected (WELHAN, 2001):

- Simulated variables can be continuous or categorical;
- Conditional simulation reproduces the observed data;
- Unconditional simulation only reproduces global statistics (histogram, variogram, etc.);
- Categoric variables can be simulated to honour specific geometric patterns (object-based);
- Continuous variables can be simulated to honour a specified covariance model;
- Variables can be simulated through optimization processes starting from scratch or from previously-simulated images, to honour additional, external, constraining data.

Gaussian Simulation, using convenience (both theoretical and practical) with which Gaussian distributions can be described and handled statistically makes a Gaussian random function model very appealing. This class of simulation algorithms is restricted to continuous variables. Gaussian simulation can be applied if the probability distribution functions (PDF's) of all variables that participate in the simulation, (e.g. a regionalized variable plus its local uncertainty or variance) are normally distributed. The most important condition is that the variables are univariate and bivariate normal and independent. The assumption of normality is not restrictive for real-world data, since a normal-score transform can be applied to make any data set normally-distributed. The most serious limitation for real data is that a regionalized variable at a given location is often not independent. It may be correlated with its local variance, (via a proportional effect) and/or it may have strong local spatial correlation, (e.g. interconnected high or low values). WELHAN (2001) indicated strong local spatial correlation as the major possible problem, and recommended a method described in DEUTSCH & JOURNEL (1997) for checking and necessary application if the Gaussian simulation approach is to be correctly defended. The proportional effect can be neglected if it is weak or is removed by moving the variogram ellipsoid (search neighborhood). Despite such caveats, the Gaussian simulation approach is widely used; its intuitive and algorithmic simplicity define its widespread appeal.

Sequential Simulation Algorithm (e.g. DEUSTSCH & JOURNEL, 1997) contains a popular class of algorithms that is in widespread use. Such an approach is very often in the Gaussian version as Sequential Gaussian Simulation, but can also be applied in simulations of other types. The sequential simulation approach can be broken down conceptually into three parts (WELHAN, 2001):

- The initialization process;
- A global random walk process and
- A local search and conditional random estimation process.

Initialization determines the univariate cumulative distribution function (CDF) from the entire study area, using declustering if necessary. A normal-score transform of the data using a standard-normal CDF must be performed, and a regular grid network of nodes defined, at which estimates will be made. Data points may be re-located to these grid nodes (for faster computation) or not (for maximum accuracy). Typically, this grid network will be much finer than was used in kriging, because simulation attempts to reproduce the nature of local variability and uncertainty.

The Global random walk process uses a random number seed. The estimation starts at a random location on a grid network with the random number generator, moves to subsequent random locations after a local search has been completed, but never visits the same grid node twice. The estimated data are considered as the new hard-data values.

Local search and conditional estimation process looks within a designated search neighborhood for a prescribed number of nearest neighbors, comprising both original data and/or previously simulated values (WELHAN, 2001). It uses the variogram model of normal-score data with Simple Kriging to estimate the mean and variance at the grid node location. This defines the cumulative distribution function (CDF) of the regionalized variable at that search neighborhood location. In the case of neither hard-data or previously simulated nodes existing within the search neighbourhood, the CDF is conditional only to the global histogram. The value estimated at such a grid node will be a value drawn at random from the CDF estimated from the global histogram (WELHAN, 2001). At the end of this process, the simulated normal-score values are transformed back into simulated values for the original variable. The described algorithm is repeated for multiple realizations, starting each time at a different initial grid location and visiting grid nodes in a different order.

The Gaussian Simulation tends to overestimate local spatial entropy, creating more apparent variability than is actually present, this being the most obvious limitation of simulation. It is also not sufficiently flexible to handle a mixture of different normal populations. Furthermore, sequential simulation does not allow the incorporation of categorical data (e.g. 1 and 0), but this is overcome by introducing an indicator simulation algorithm. The Gaussian sequential simulation approach is widely used despite these limitations.

Anyone interested in geostatistical theory, both deterministic or stochastic, as applied in the Croatian part of the Pannonian Basin is advised to follow the research presented in MALVIĆ (2003, 2005, 2006) or SMOLJANOVIĆ & MALVIĆ (2005).

4. EXAMPLE 1: COKRIGING OR STOCHASTIC CO-SIMULATION

The Beničanci field is the largest hydrocarbon reservoir in the Beničanci oil zone, and the largest oil producing field in the Croatian part of the Drava Depression (**Figure 1**). The oil reservoir is located in breccia of Badenian age, and is overlain by structurally shallower and smaller gas reservoirs. This field and the entire zone located in the eastern part of the Drava Depression, has been studied for decades. The different geological and petroleum settings have been published in many papers. ŠIMON & BATUŠIĆ (1974) described the lithostratigraphic section of the Beničanci field. TURAJLIĆ et al. (1979) compared the development of the Beničanci, Obod and Ladislavci-Kučanci fields. HERNITZ et al. (1993, 1995) described shallow onshore clastics of Miocene age and source rocks in the wider area of the Beničanci field, as well as in other parts of the Drava Depression.

The depositional model of Neogene clastics was in detail described by TIŠLJAR (1993). Structurally, the field is defined by brachianticline axes of 8.0 by 1.3 km, faulted mostly by displacements characterised by a NW-SE strike. However, the main reverse fault closes the structure in the south (**Figure 2**). The entire field is an anticlinorium that includes four smaller anticlines, sinking toward the east.

The stratigraphy includes basements rocks of Permian and Triassic age and Neogene clastic sediments. Different basement schists are determined only in wells deeper than 2500 metres. The Miocene stratigraphy is very heterogeneous. The oldest rocks are sporadically represented by effusives belonging to the Miocene magmatic cycle, which have been partially re-deposited. Their lateral equivalents are Triassic limy breccia and quartzites that have also been redeposited. The main sedimentary sequence is of Badenian age, represented by dolomitic and limestone breccia. Detritus is mostly dolomitic and can reach metre dimensions. The matrix is also of micro to crypto dolomite, tectonically crushed, petrified and crystallized. Reservoir carbonate breccias are characterised by relatively low primary porosity, but secondary processes (including dissolution, faulting and fracturing) resulted in increased secondary porosity. The shallow part of the reservoir is characterised by a mixture of breccia and conglomerate, with mixed carbonate and siliciclastic detritus. In general, Badenian breccias were deposited in a shallow marine environment, and consequently could be re-deposited and weathered by currents and wave fronts. TIŠLJAR (1993) described this Badenian sedimentary sequence through four types of possible reservoirs as follows:

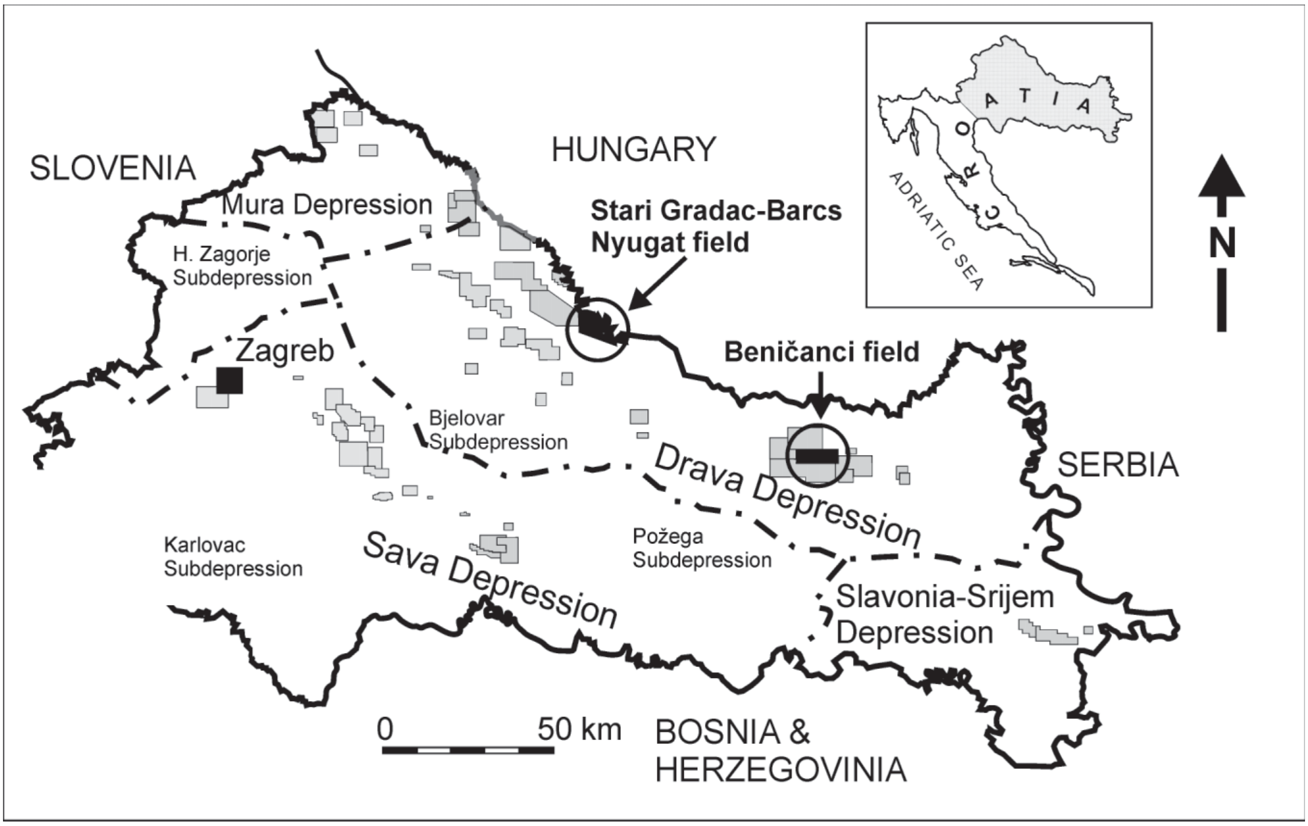

Figure 1: Location map of the Beničanci and Stari Gradac-Barcs Nyugat fields in Drava Depression

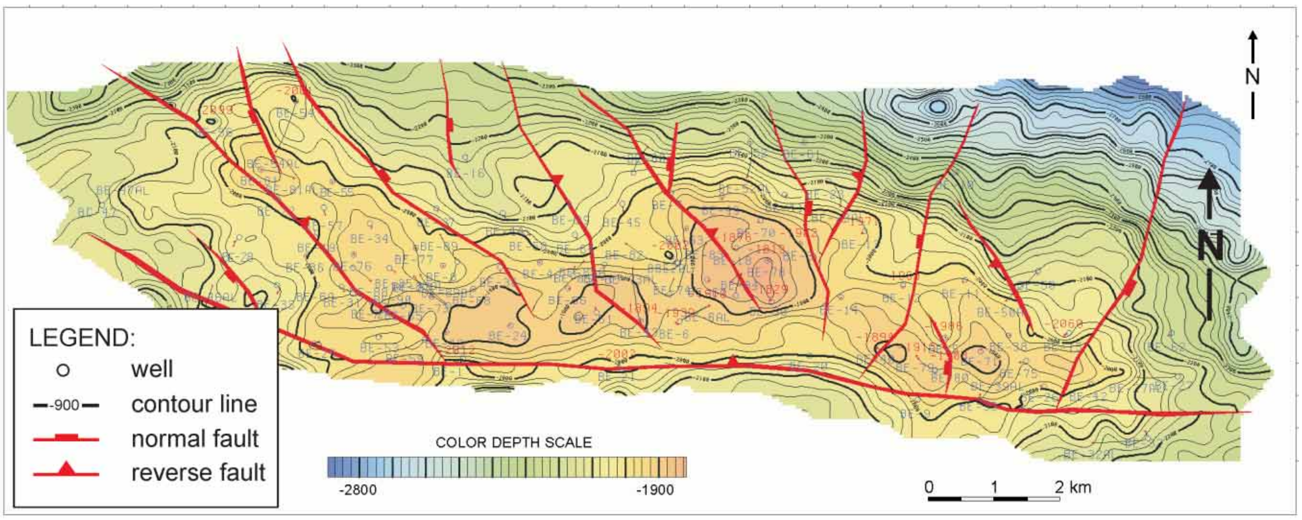

Figure 2: Palaeorelief map of the Beničanci field (after FUTIVIĆ & PLEIĆ, 2002)

1. Clinoform bodies of carbonate rockfall and debrite breccias;
2. Nearshore conglomerates;
3. Channel and fan bodies made of breccia-conglomerates and sandstones.
4. Tectonic breccias, generated by crushing and tectonization of previously lithified Miocene turbidite sedimetary rocks.

The observed reservoir mostly belongs to type 4 and partially to type 1.

Badenian sediments were overlain by a typical, younger sedimentary sequence in the Drava depression. This began with Lower Pannonian calcitic marls (»Croatica beds«), which were eroded, on the southern part of the field. Then, Late Pannonian sediments (»Banatica beds«) of similar lithology were deposited, which are interestingly characterised by greater thicknesses in the southern part of the structure. Early Pontian sediments (»Abichi beds«) are mostly sandstones (mixture detritus) intercalated with hard marls. Late Pontian (»Rhomboidea beds«) sediments are mostly weak sandstones and medium to weak marls, sometimes clayey, including coal. The youngest sediments of Pliocene age are mostly unconsolidated clay and sand, as well as Quaternary gravel, sand and clay with limestone concretions.

The target, or primary, variable that had been geostatistically analyzed in the breccia reservoir of Badenian age was the average reservoir porosity. Fourteeen locations (or wells) were chosen as the most representative hard-data points, derived from the newest log-curve analyses. These values are mostly regularly distributed across the reservoir area, and can be observed on the porosity maps.

Moreover, the analysed reservoir is also covered by 3D seismic cube and interpreted attributes. Attribute analysis was targeted for the interval that represents the reservoir defined by permeable lithology. It begins 20 m from the seal rock in the youngest Badenian sediments and continues to either the reservoir base defined by Palaeozoic basement rocks, or the base of the well where the basement rocks were not drilled. Seismic attribute analysis encompasses amplitude attributes, frequency and phase.

The existence of seismic attributes made application of geostatistical interpolation of the cokriging possible. The cokriging requires the determination of an additional, secondary variable that describes the behaviour of the primary one. Six attributes were correlated with porosity, searching for a connection. Unfortunately, the values of the *Pearson correlation coefficient* did not show any significant correlation, due to the absence of a Gaussian distribution in the analyzed variables. It was more appropriate to use the non-parametric *Spearman rank correlation coefficient*, based on a median value (instead of normal distribution parameters like standard deviation and mean). The highest correlation was calculated for the pair porosity-reflection strength (as a derivative of amplitude) where the value of r' = – 0.64.

The relevant variogram model had been defined along field structural axes. The direction of the primary variogram axis was 90-270^{o} (E-W), while the secondary spreads occurred along the direction 0-180^{o}. Due to the low number of data, the angle tolerance was set at a maximum value of 45^{o}. The ranges are almost isotropic – for the main axis the range is 1750 metres determined from 7 or more data pairs. The range of the secondary axis is 1500 metres, determined subjectively considering only 5 or less data pairs, and consequently only 4 existing points. Experimental variograms had been approximated by spherical theoretical models.

The **cokriging** results are presented in **Figure 3b**. The selected technique was collocated cokriging. The word »collocated« means that the primary and secondary variables were observed at the same locations, i.e. the secondary variable had been extrapolated at the same locations where the hard-data exist. The cokriging results were significantly better and more descriptive in the inter-well areas than the kriging results (MALVIĆ & ĐUREKOVIĆ, 2003). This could easily be concluded from results of the cross-validation method (DAVIS, 1987), where the cokriging map was characterised by a mean square error of 2.19 and the kriging map by the value of 2.97.

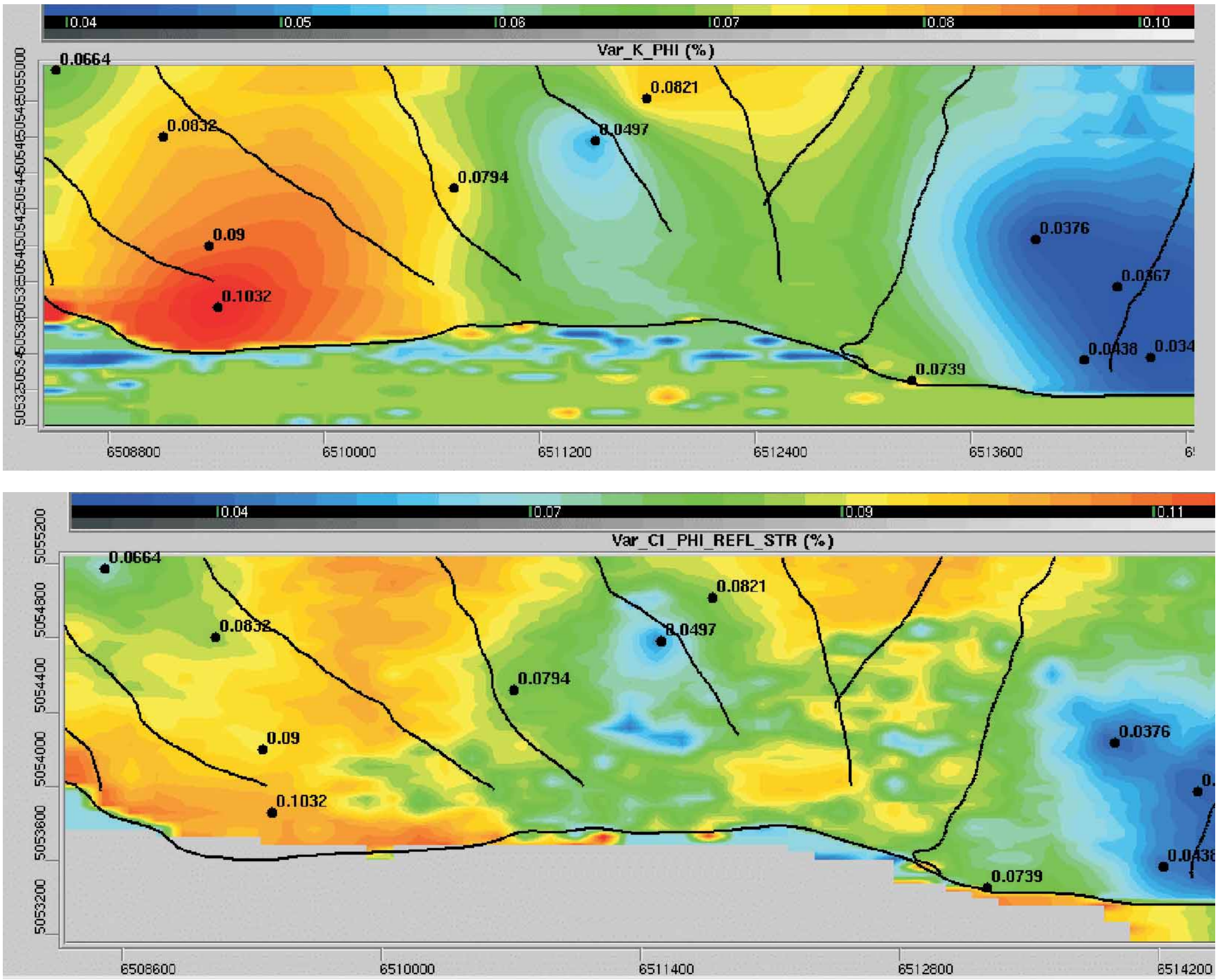

Figure 3: Kriging (Figure 3a – above) and cokriging (Figure 3b – below) maps of porosity distribution

Although the total difference between the kriging and cokriging maps (**Figure 3a and 3b**), in a numerical sense, is about 25%, this difference is mostly regularly distributed across the entire reservoir area.

The secondary approach for porosity distribution at the same reservoir included using **Sequential Gaussian Simulations** (e.g. DUBRULE, 1998 and KELKAR & PEREZ, 2002), as an approach that could include the majority of uncertainties connected to the input data and inter-well area, especially in zones distant from hard-data. A set of only four stochastical equally probable realizations were calculated, which were found to express the noisy nature of the hard-data. Generally, there must be at least 100-200 realizations for acceptable measurement of the degree of uncertainty. These realizations were based on a cokriging map as »zero« realization.

Due to stochastic theory, data were normalized to Gaussian distributions and their PDF curve was used to sample new porosity values. The new, data set kept the same statistics characteristic as the input data set. Each cell has a porosity value sampled from its own normally shaped PDF defined with mean and variance values characteristic for the cokriging »zero« realization (e.g. DEUTSCH, 2002). The order of the simulated cells was random and sequential.

The »extreme« realization P1 and P99 are shown on **Figure 4**. The symbol P1 means that 99 % of all other possible realizations would describe larger total reservoir porosity. The meaning of P99 realization is the opposite. The comparison by cokriging map, or zero »solution«, reveals that the P99 realization looks very similar to the cokriging map. Furthermore, it can be concluded that the cokriging map is a very good representation of porosity distribution, and the stochastic solution suggests that mapped uncertainties in the inter-well area can be considered pretty homogeneous for the entire statistical range.

Due to the fact that Gaussian simulation overestimates local variability, together with the observed similarity between the deterministic and stochastic realizations in the case of the existing seismic attribute and limited dataset (about 15 hard-data), the cokriging approach is to be recommended rather than the use of Gaussian simulations.

5. EXAMPLE 2: KRIGING OR STOCHASTIC SIMULATION

The Stari Gradac-Barcs Nyugat gas-condensate field is located on the Croatian-Hungarian border, along the Drava River, approximately 150 km east of Zagreb. The field is situated in

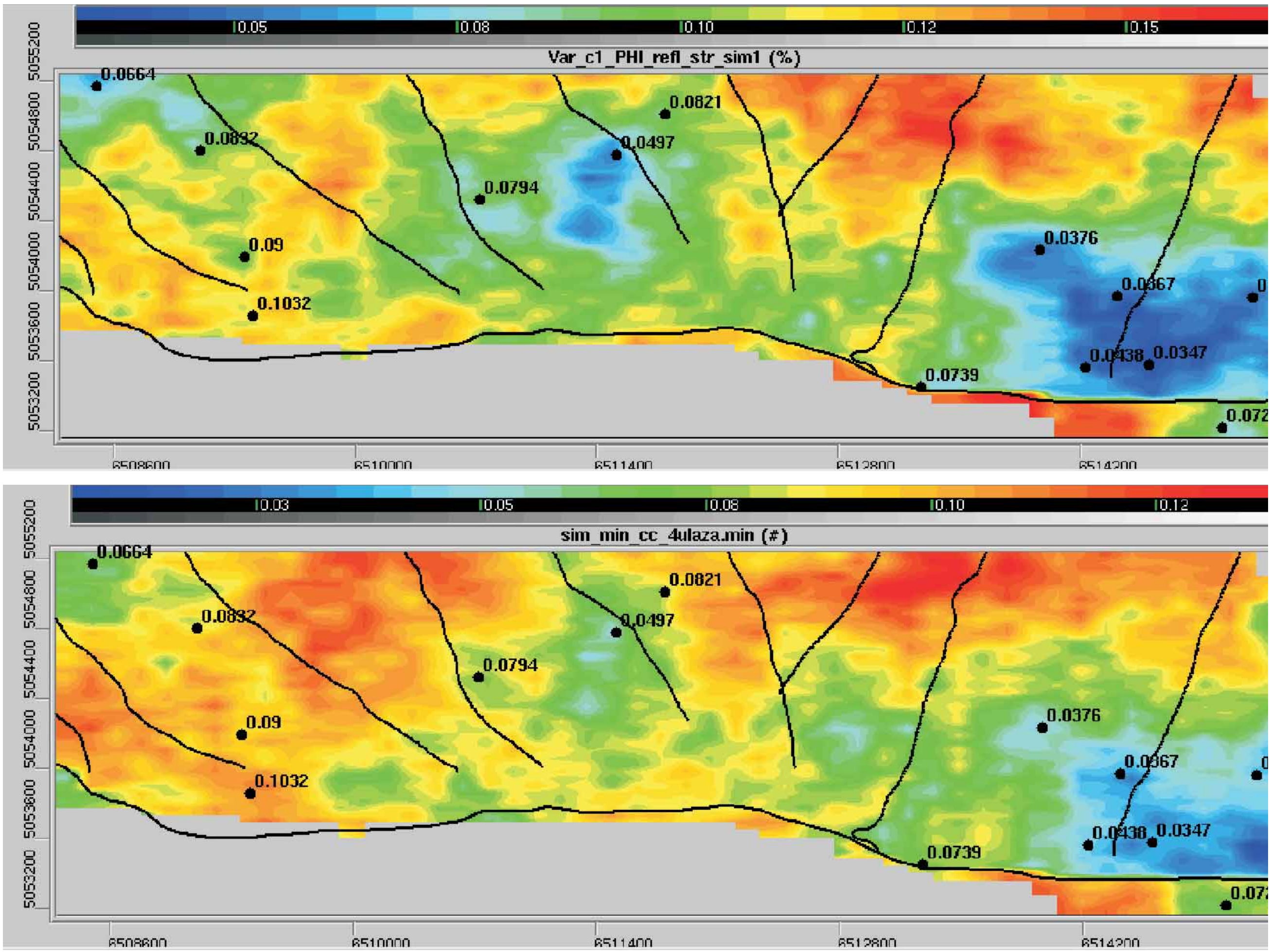

Figure 4: P1 (Figure 4a – above) and P99 (Figure 4b – below) equiprobable stochastic realizations

the Northwestern part of the Drava depression (**Figure 1**). A massive (anticline) reservoir is trapped by a combined structural-stratigraphic closure (**Figure 5**). The lithology of the reservoir is very complex, divided into four lithofacies: 'Clastites' of Badenian age, 'Dolomites' of Lower Triassic age, 'Quartzites' of Lower Triassic age and 'Metavolcanites' of Permian, Devonian and (possibly) Carboniferous age.

The results of geostatistical porosity mapping in lithofacies 'Clastites' of Badenian age that included mostly carbonate breccia are analysed here. These reservoir lithofacies are lithologically comparable to coarse-grained Badenian lithofacies analysed from the Beničanci field.

The input dataset was relatively small in number, which consequently excluded the existing arithmetic mean as being valid for a global mean (i.e. excluded condition of first-order stationarity necessary for Simple Kriging). Ordinary Kriging was selected as the most appropriate interpolation technique. There was no established correlation between seismic attributes and porosity.

The variogram analysis was done again along the main structural axes of the field (**Figure 5**). The principal variogram axis has a strike of 120-300° and the subordinate strike is 30-210°. The experimental variograms were calculated for each lithofacies, and particularly for the 'Clastites'. The ranges are 3500 metres (principal) and 2000 metres (subordinate axis). The principal axis modelling is fairly reliable based on 8 or more data pairs. The modelling of the secondary axis is highly subjective. All variograms are again approximate by spherical theoretical models.

The kriging map shows a significant interpolation improvement compared to those previously produced by Inverse Distance Weighting (**Figure 6**). If these two solutions are compared numerically through cross-validation results, the improvement is more than 30% – for kriging map 3.91 and for Inverse Distance Weighting map 5.28.

It is not difficult to explain why the kriging map is more accurate and acceptable for the geological model. It is clear that the Inverse Distance Weighting map (**Figure 6, left**) is unrealistically too detailed. Based on only 14 hard-data points, the smoothed kriging map (**Figure 6, right**) is more realistic and interpretable. However, the challenge is to try to describe inter-well area uncertainties by stochastic simulation as well. Using respected hard-data points, i.e. conditional simulations, 100 simulations were calculated describing reservoir randomness (**Figure 7**).

The reservoir space is always characterised by uncertainties. The permanent problem is how to express these »hidden« characteristics in the mapped variable. This is especially emphasised in the case of a very limited input dataset. In this case, for the 'Clastites' lithofacies at the Stari Gradac-Barcs Nyu-

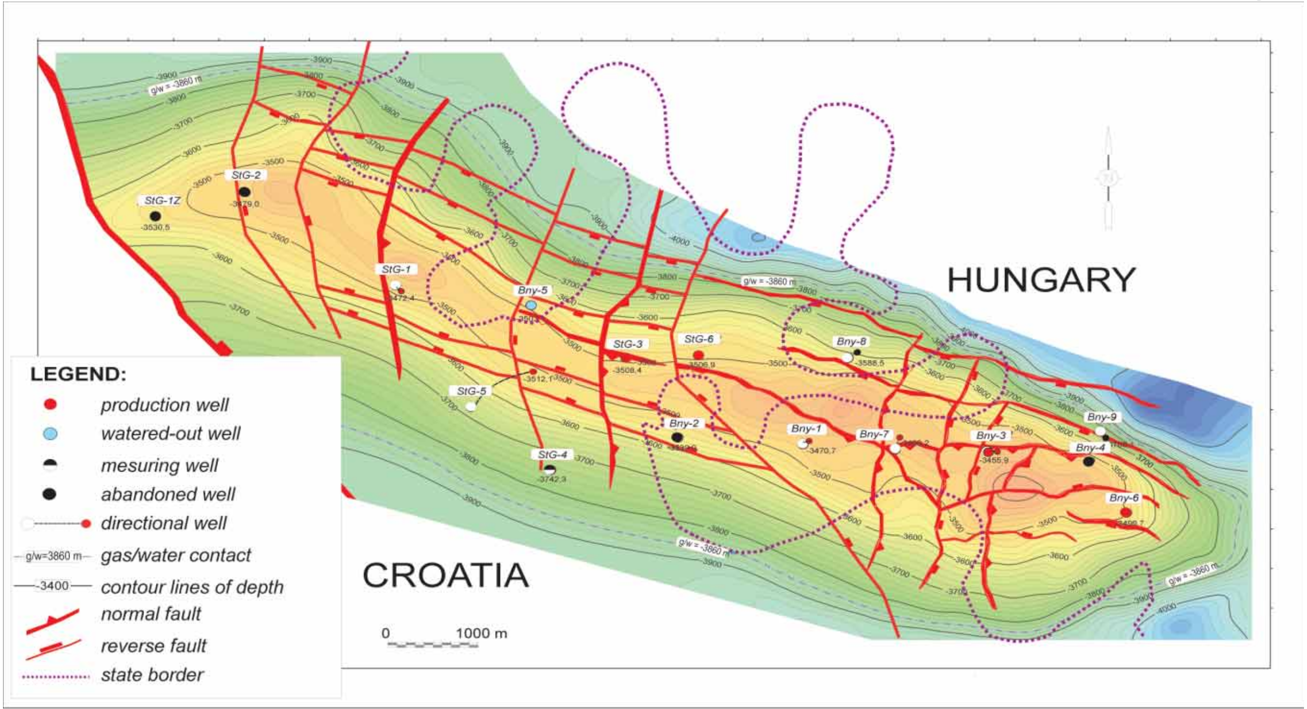

Figure 5: Structure map of the top of the 'Clastites' lithofacies (after GAĆEŠA at al., 2001[3])

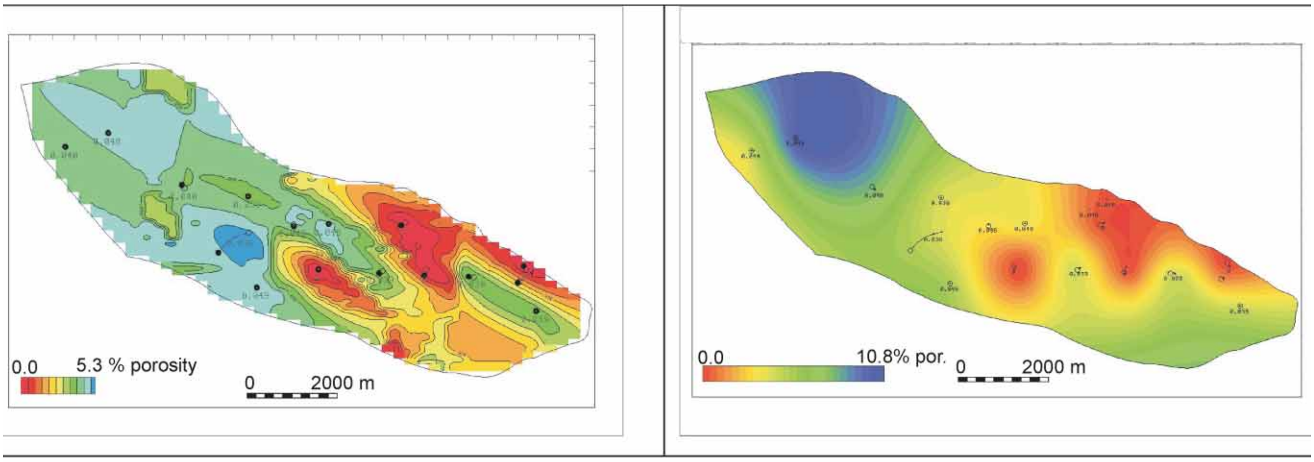

Figure 6: Schematic porosity maps interpolated by Inverse Distance Weighting (left) and kriging (right)

gat field, the standard deviation of input porosity dataset was higher than 0.5 of the possible maximum. The statistical uncertainty is mostly included in the variogram model, forcing the large primary range and estimated range of the subordinate axis from the ratio of the structural axes. These facts just indicate a stochastic approach as the most favourable for inter-well area description. The differences in the set of simulated realizations (difference between P1 and P99) were about 14% (or minimum volume of 'Clastites' was 5,241,097 and maximum 5,973,280 m^3). Moreover, it is important to emphasis that both kriging and stochastic realizations led to a decrease in reservoir volume, over that previously calculated by Inverse Distance Weighting. In any case, where the input dataset is very limited, with the absence of an additional, significantly correlated seismic variable, the stochastic approach is preferred.

The Stari Gradac-Barcs Nyugat dataset is very limited and it is hard to describe local variability. A stochastic approach is more descriptive and allows geologist to speculate about features that influence porosity distribution. Finally it led to better characterisation of porosity in the inter-well area.

6. CONCLUSIONS

Porosity interpolation results were compared at two similar reservoirs from the Beničanci and Stari Gradac-Barcs Nyugat fields. Both are located in the Drava Depression, and occur in coarse-grained sediments (carbonate breccia) of Badenian age. The research goal was to establish qualitative criteria to aid the

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[2] GAĆEŠA, S., FUTIVIĆ, I, FERENCZ, G. & HORVATH, Z. (2001): Barcs Nyugat-Stari Gradac field; Geological evaluation-summary study. Unpublished report, company internal files, INA Naftaplin, Sektor za razradu, 27 p.

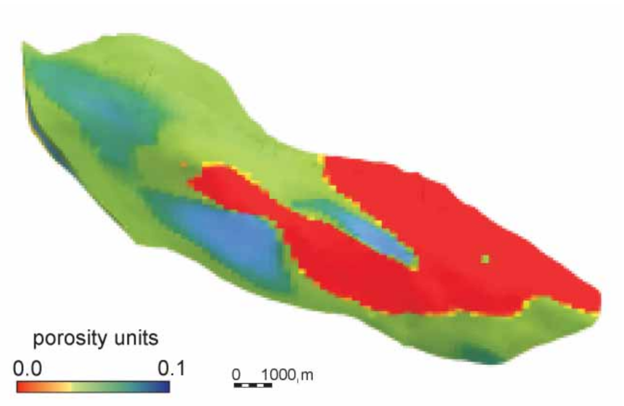

Figure 7: Porosity estimated in median stochastic realization (P50)

selection of either deterministic or stochastic geostatistical approaches for porosity interpolation in analyzed reservoirs.

The first case included use of an additional seismic variable and consequently the cokriging method. The second example was oriented only on a primary variable interpolated by kriging and comparison of stochastical realizations using the kriging map.

In the first case, i.e. porosity analysis at the Beničanci field, seismic data led to a lower estimation error (cokriging variance error) and fortunately this advantage was regularly distributed across the entire reservoir. Moreover, the seismic attribute attenuated the ‘bull-eye-effect’, so obviously present on the kriging map. The cokriging map can be considered as a very reliable representation of the distribution of porosity in the breccia reservoir in the Beničanci field. The set of stochastic realization results only confirmed that the cokriging map can be used as a ‘definite’ porosity solution, very optimistically describing the reservoir, (the cokriging map is very similar to the stochastic map obtained by P99 realization). Moreover, the cokriging realization will not tend to over-emphasis local variability, which could be characteristic of co-simulation.

The second case, analysis at the Stari Gradac-Barcs Nyugat field, was different because it was not possible to select a secondary seismic source of information. Many of the uncertainties were again incorporated in a variogram model, in a similar way to the Beničanci reservoir. However, the Stari Gradac-Barcs Nyugat dataset was characterised by a very high standard deviation, although there were only 14 hard-data points. It means that the mean and variance can not be considered as representative, and randomness is one of the most obvious reservoir characteristics. It strongly indicates a stochastic approach for testing and calculation of a relatively wide set of realizations (100 maps were calculated). Both approaches, the kriging and Sequential Gaussian Simulations, yielded better solutions than those obtained by the Inverse Distance Weighting method. Moreover, the stochastic approach led to better reservoir characterization, because it depicted much more variation in the zones distant from hard-data than was visible on the kriging map. Comparing P1, P50 and P99 realizations only the border of zone of low porosity (red zone shown at **Figure 7**) and consequently of high porosity (light blue area at **Figure 7**) can be predicted. This zone also indicates the boundary between better and poorer parts of the reservoir, i.e. at the boundary between proximal and distal parts of the alluvial fan, where coarse-grained clastics of Badenian age were deposited (MALVIĆ, 2006).

Finally, both examples represent very limited datasets of 14 hard-data values. The results indicated that if a secondary source of information (like seismic) is available then the deterministic cokriging approach is recommended. However, if the secondary variable is not available the stochastic Gaussian simulation is a more descriptive tool.

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